ELECTROPRODUCTION OF A LIGHT NEUTRAL VECTOR MESON AT NEXT-TO-LEADING ORDER

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Abstract

The process of a light neutral vector meson electroproduction is studied in the framework of QCD factorization in which the amplitude factorizes in a convolution of the nonperturbative meson distribution amplitude and the generalized parton densities with the perturbatively calculable hard-scattering amplitudes. We derive a complete set of hard-scattering amplitudes at next-to-leading order (NLO) for the production of vector mesons, $V = \rho^0, \omega, \phi$.

1. The process of elastic neutral vector meson electroproduction on a nucleon,

$$\gamma^*(q)\, N(p) \to V(q')\, N(p') \ , \quad \text{where} \quad V = \rho^0,\, \omega,\, \phi \,, \tag{1}$$

was studied in fix target and in HERA collider experiments. The primary motivation for the strong interest in this process (and in the similar process of heavy quarkonium production) is that it can potentially serve to constrain the gluon density in a nucleon [1, 2]. On the theoretical side, the large negative virtuality of the photon, $q^2 = -Q^2$, provides a hard scale for the process which justifies the application of QCD factorization methods that allow to separate the contributions to the amplitude coming from different scales. The factorization theorem [3] states that in a scaling limit, $Q^2 \to \infty$ and $x_{Bj} = Q^2/2(p\cdot q)$ fixed, a vector meson is produced in the longitudinally polarized state by the longitudinally polarized photon and that the amplitude of the process (1) is given by a convolution of the nonperturbative meson distribution amplitude (DA) and the generalized parton densities (GPDs) with the perturbatively calculable hard-scattering amplitudes. In this contribution we present the results of our calculation of the hard-scattering amplitudes at NLO.

2. $p^2 = p'^2 = m_N^2$ and $q'^2 = m_M^2$, where m_N and m_M are a proton mass and a meson mass respectively. The invariant c.m. energy $s = (q+p)^2 = W^2$. We define

$$\Delta = p' - p \,, \quad P = \frac{p+p'}{2} \,, \quad t = \Delta^2 \,,$$
$$(q-\Delta)^2 = m_M^2 \,, \quad x_{Bj} = \frac{Q^2}{W^2+Q^2} \,. \tag{2}$$

We introduce two light-cone vectors

$$n_+^2 = n_-^2 = 0 \,, \quad n_+ n_- = 1 \,. \tag{3}$$

Any vector a is decomposed as

$$a^\mu = a^+ n_+^\mu + a^- n_-^\mu + a_\perp\,, \quad a^2 = 2\,a^+ a^- - \vec{a}^2\,. \tag{4}$$

We choose the light-cone vectors in such a way that

$$\begin{aligned} p &= (1+\xi)W n_+ + \frac{m_N^2}{2(1+\xi)W}\, n_-\,, \\ p' &= (1-\xi)W n_+ + \frac{(m_N^2+\vec{\Delta}^2)}{2(1-\xi)W}\, n_- + \Delta_\perp\,. \end{aligned} \tag{5}$$

We are interested in the kinematic region where the invariant transferred momentum,

$$t = -\left(\frac{4\,\xi^2}{1-\xi^2} m_N^2 + \frac{1+\xi}{1-\xi}\vec{\Delta}^2\right)\,, \tag{6}$$

is small, much smaller than Q^2. In the scaling limit variable ξ which parametrizes the plus component of the momentum transfer equals $\xi = x_{Bj}/(2-x_{Bj})$.

GPDs are defined as the matrix element of the light-cone quark and gluon operators:

$$\begin{aligned} & F^q(x,\xi,t) = \frac{1}{2}\int \frac{d\lambda}{2\pi} e^{ix(Pz)} \langle p'|\bar{q}\left(-\frac{z}{2}\right) \not{n}_- q\left(\frac{z}{2}\right)|p\rangle|_{z=\lambda n_-} \\ & = \frac{1}{2(Pn_-)}\left[\mathcal{H}^q(x,\xi,t)\,\bar{u}(p')\not{n}_- u(p) + \mathcal{E}^q(x,\xi,t)\,\bar{u}(p')\frac{i\sigma^{\alpha\beta}n_{-\alpha}\Delta_\beta}{2\,m_N}u(p)\right]\,, \end{aligned} \tag{7}$$

$$\begin{aligned} & F^g(x,\xi,t) = \frac{1}{(Pn_-)}\int \frac{d\lambda}{2\pi} e^{ix(Pz)}\, n_{-\alpha} n_{-\beta}\, \langle p'|G^{\alpha\mu}\left(-\frac{z}{2}\right) G^\beta_\mu\left(\frac{z}{2}\right)|p\rangle|_{z=\lambda n_-} \\ & = \frac{1}{2(Pn_-)}\left[\mathcal{H}^g(x,\xi,t)\,\bar{u}(p')\not{n}_- u(p) + \mathcal{E}^g(x,\xi,t)\,\bar{u}(p')\frac{i\sigma^{\alpha\beta}n_{-\alpha}\Delta_\beta}{2\,m_N}u(p)\right]\,. \end{aligned} \tag{8}$$

In both cases the insertion of the path-ordered gauge factor between the field operators is implied. Momentum fraction x, $-1 \le x \le 1$, parametrizes parton momenta with respect to the symmetric momentum $P = (p+p')/2$. In the forward limit, $p' = p$, the contributions proportional to the functions $\mathcal{E}^q(x,\xi,t)$ and $\mathcal{E}^g(x,\xi,t)$ vanish, the distributions $\mathcal{H}^q(x,\xi,t)$ and $\mathcal{H}^g(x,\xi,t)$ reduce to the ordinary quark and gluon densities:

$$\begin{aligned} &\mathcal{H}^q(x,0,0) = q(x) \quad \text{for} \quad x>0\,, \\ &\mathcal{H}^q(x,0,0) = -\bar{q}(-x) \quad \text{for} \quad x<0\,; \\ &\mathcal{H}^g(x,0,0) = x\,g(x) \quad \text{for} \quad x>0\,. \end{aligned} \tag{9}$$

Note that gluon GPD is even function of x, $\mathcal{H}^g(x,\xi,t) = \mathcal{H}^g(-x,\xi,t)$.

The meson DA $\phi_V(z)$ is defined by the following relation

$$\langle 0|\bar{q}(y)\gamma_\mu q(-y)|V_L(p)\rangle_{y^2\to 0} = p_\mu\, f_V \int_0^1 dz\, e^{i(2z-1)(py)}\phi_V(z)\,. \tag{10}$$

It is normalized to unity $\int_0^1 \phi_V(z)dz = 1$. Here z is a light-cone fraction of a quark, f_V is a meson dimensional coupling constant known from $V \to e^+e^-$ decay, $f_\rho = 198 \pm 7\,\mathrm{MeV}$.

$$\mathcal{M}_{\gamma_L^* N\to V_L N} = \frac{2\pi\sqrt{4\pi\alpha}\, f_V}{N_c\, Q\, \xi} \int\limits_0^1 dz\, \phi_V(z) \int\limits_{-1}^1 dx \Big[\tag{11}$$

$$Q_V \left(T_g(z,x)\, F^g(x,\xi,t) + T_{(+)}(z,x) F^{(+)}(x,\xi,t)\right) + \sum_q e_q^V T_q(z,x) F^{q(+)}(x,\xi,t)\Big] \,.$$

Here the dependence of the GPDs, DA and the hard-scattering amplitudes on factorization scale μ_F is suppressed for shortness. Since we consider the leading helicity non-flip amplitude, in eq. (11) the hard-scattering amplitudes do not depend on t. The account of this dependence would lead to the power suppressed, $\sim t/Q$, contribution. α is a fine structure constant, $N_c = 3$ is the number of QCD colors. Q_V depends on the meson flavor content. If one assumes it is $\frac{1}{\sqrt{2}}(|u\bar{u}\rangle - |d\bar{d}\rangle)$, $\frac{1}{\sqrt{2}}(|u\bar{u}\rangle + |d\bar{d}\rangle)$ and $|s\bar{s}\rangle$ for ρ, ω and ϕ respectively, than $Q_\rho = \frac{1}{\sqrt{2}}$, $Q_\omega = \frac{1}{3\sqrt{2}}$ and $Q_\phi = \frac{-1}{3}$, the sum in the last term of (11) runs over $q = u, d$ for $V = \rho, \omega$ and $q = s$ for $V = \phi$ and

$$e_u^\rho = e_u^\omega = \frac{2}{3\sqrt{2}}\,, \quad e_d^\rho = -e_d^\omega = \frac{1}{3\sqrt{2}} \quad e_s^\phi = \frac{-1}{3}\,.$$

$F^{q(+)}(x,\xi,t) = F^q(x,\xi,t) - F^q(-x,\xi,t)$ denotes a singlet quark GPD, $F^{(+)}(x,\xi,t) = \sum_{q=u,d,s} F^{q(+)}(x,\xi,t)$ stands for the sum of all light flavors.

Due to odd C- parity of a vector meson $\phi_V(z) = \phi_V(1-z)$. Moreover, since V and γ^* have the same C- parities, $\gamma^* \to V$ transition selects the C-even gluon and singlet quark contributions, whereas the C-odd quark combination $F^{q(-)}(x,\xi,t) = F^q(x,\xi,t) + F^q(-x,\xi,t)$ decouples in (11).

3. Below we present the results of our calculation of the hard-scattering amplitudes in the $\overline{\mathrm{MS}}$ scheme.

$T_q(z,x)$ may be obtained by the following substitution from the known results for a pion EM formfactor, see also [4],

$$T_q(z,x) = \left\{ T\left(z, \frac{x+\xi}{2\xi} - i\epsilon\right) - T\left(\bar{z}, \frac{\xi - x}{2\xi} - i\epsilon\right)\right\} + \{z \to \bar{z}\}\,, \tag{12}$$

$$\begin{aligned} T(v,u) = {} & \frac{\alpha_S(\mu_R^2) C_F}{4vu} \left(1 + \frac{\alpha_S(\mu_R^2)}{4\pi}\left[c_1 \left(2\,[3 + \ln(vu)] \ln\left(\frac{Q^2}{\mu_F^2}\right) + \ln^2(vu)\right.\right.\right. \\ & \left. + 6\ln(vu) - \frac{\ln(v)}{\bar{v}} - \frac{\ln(u)}{\bar{u}} - \frac{28}{3}\right) + \beta_0\left(\frac{5}{3} - \ln(vu) - \ln\left(\frac{Q^2}{\mu_R^2}\right)\right) \\ & + c_2 \left(2\frac{(\bar{v}v^2 + \bar{u}u^2)}{(v-u)^3}\left[Li_2(\bar{u}) - Li_2(\bar{v}) + Li_2(v) - Li_2(u) + \ln(\bar{v})\ln(u) - \ln(\bar{u})\ln(v)\right]\right. \\ & + 2\left[Li_2(\bar{u}) + Li_2(\bar{v}) - Li_2(u) - Li_2(v) + \ln(\bar{v})\ln(u) + \ln(\bar{u})\ln(v)\right] \\ & \left.\left.\left. + 4\frac{vu\ln(vu)}{(v-u)^2} + 2\frac{(v+u-2vu)\ln\bar{v}\bar{u}}{(v-u)^2} - 4\ln(\bar{v})\ln(\bar{u}) - \frac{20}{3}\right)\right]\right)\,. \end{aligned} \tag{13}$$

Here and below we use a shorthand notation $\bar{u} = 1 - u$ for any light-cone fraction. μ_R is a renormalization scale for a strong coupling, $\beta_0 = \frac{11N_c}{3} - \frac{2n_f}{3}$, n_f is the effective number

of quark flavors. $C_F = \frac{N_c^2-1}{2N_c}$, $Li_2(z) = -\int\limits_0^z \frac{dt}{t}\ln(1-t)$. Also we denote

$$c_1 = C_F\,, \quad c_2 = C_F - \frac{C_A}{2} = -\frac{1}{2N_c}\,. \tag{14}$$

$T_{(+)}(z,x)$ starts from NLO

$$T_{(+)}(z,x) = \frac{\alpha_S^2(\mu_R^2)\,C_F}{(8\pi)z\bar z}\,\mathcal{I}_q\left(z, \frac{x-\xi}{2\xi}+i\epsilon\right)\,, \tag{15}$$

here

$$\begin{aligned}\mathcal{I}_q(z,y) = &\left\{\frac{2y+1}{y(y+1)}\left(\left(\ln\Big(\frac{Q^2}{\mu_F^2}\Big)-1+\ln(z)\right)\Big(y\ln(-y)-(y+1)\ln(y+1)\Big)\right.\right.\\ &\left.+\frac{y}{2}\ln^2(-y)-\frac{y+1}{2}\ln^2(y+1)\right)+\frac{y\ln(-y)+(y+1)\ln(y+1)}{y(y+1)}\\ &\left.-\frac{R(z,y)}{y+z}+\frac{y(y+1)+(y+z)^2}{(y+z)^2}\,H(z,y)\right\}+\{z\to\bar z\}\,,\end{aligned} \tag{16}$$

where we introduced two auxiliary functions

$$R(z,y) = z\ln(-y)+\bar z\ln(y+1)+z\ln(z)+\bar z\ln(\bar z)\,, \tag{17}$$

$$H(z,y) = Li_2(y+1)-Li_2(-y)+Li_2(z)-Li_2(\bar z)+\ln(-y)\,\ln(\bar z)-\ln(y+1)\,\ln(z)\,. \tag{18}$$

For the gluonic contribution we obtain

$$T_g(z,x) = \frac{\alpha_S(\mu_R^2)\,\xi}{z\bar z\,(x+\xi-i\epsilon)(x-\xi+i\epsilon)}\left[1+\frac{\alpha_S(\mu_R^2)}{4\pi}\,\mathcal{I}_g\left(z,\frac{x-\xi}{2\xi}+i\epsilon\right)\right]\,, \tag{19}$$

where

$$\begin{aligned}\mathcal{I}_g(z,y) = &\left\{-\frac{\beta_0}{2}\left(\ln\Big(\frac{Q^2}{\mu_R^2}\Big)-1\right)\right.\\ &+\left(\ln\Big(\frac{Q^2}{\mu_F^2}\Big)-1\right)\left[\frac{c_1}{2}\left(\frac{y\ln(-y)}{y+1}+\frac{(y+1)\ln(y+1)}{y}\right)+c_1\left(\frac{3}{2}+2z\ln(\bar z)\right)\right.\\ &\left.+\frac{2(c_1-c_2)(y^2+(y+1)^2)}{y(y+1)}\Big(y\ln(-y)-(y+1)\ln(y+1)\Big)+\frac{\beta_0}{2}\right]\\ &+(c_1-c_2)(2y+1)\Big(\ln(-y)-\ln(y+1)\Big)\left(\frac{3}{2}+\ln(z\bar z)+\ln(-y)+\ln(y+1)\right)\\ &+\Big(c_1(y(y+1)+(y+z)^2)-c_2(2y+1)(y+z)\Big)\left[\frac{y(y+1)+(y+z)^2}{(y+z)^3}H(z,y)\right.\\ &\left.-\frac{R(z,y)}{(y+z)^2}+\frac{\ln(-y)-\ln(y+1)+\ln(z)-\ln(\bar z)}{2(y+z)}\right]-\frac{c_1(2y+1)R(z,y)}{2(y+z)}-2c_1\\ &-(c_1-c_2)\Big(\ln(z\bar z)-2\Big)\left(\frac{y\ln(-y)}{y+1}+\frac{(y+1)\ln(y+1)}{y}\right)+c_1(1+3z)\ln(\bar z)\\ &-\frac{(3c_1-4c_2)}{4}\left(\frac{y\ln^2(-y)}{y+1}+\frac{(y+1)\ln^2(y+1)}{y}\right)+c_1 z\ln^2(\bar z)\\ &\left.+\Big(\ln(-y)+\ln(y+1)\Big)\left(c_1\left(\bar z\ln(z)-\frac{1}{4}\right)+2c_2\right)\right\}+\{z\to\bar z\}\,.\end{aligned} \tag{20}$$

4. Above equations give a complete description of a neutral vector meson electroproduction with NLO accuracy. At leading order we reproduce known result [5], our results for the NLO correction are new.

At high energies, $W^2 \gg Q^2$, the imaginary part of the amplitude dominates. Leading contribution to the NLO correction comes from the integration region $\xi \ll |x| \ll 1$, simplifying the gluon hard-scattering amplitude in this limit we obtain the estimate

$$\mathcal{M}_{\gamma_L^* N \to V_L N} \approx \frac{-2\, i\, \pi^2 \sqrt{4\pi\alpha}\, \alpha_S f_V Q_V}{N_c\, Q\, \xi} \tag{21}$$
$$\int\limits_0^1 \frac{dz\, \phi_V(z)}{z\bar{z}} \left[F^g(\xi,\xi,t) + \frac{\alpha_S N_c}{\pi} \ln\left(\frac{Q^2 z\bar{z}}{\mu_F^2}\right) \int\limits_\xi^1 \frac{dx}{x} F^g(x,\xi,t) \right] .$$

Given the behavior of the gluon GPD at small x, $F^g(x,\xi,t) \sim const$, we see that NLO correction is parametrically large and negative unless one chooses the value of the factorization scale sufficiently lower than the kinematic scale. For the asymptotic form of meson DA, $\phi_V^{as}(z) = 6z\bar{z}$, the last term in (21) changes the sign at $\mu_F = \frac{Q}{e}$, for the DA with a more broad shape this happens at even lower values of μ_F.

Acknowledgments

Work of D.I. is supported in part by Alexander von Humboldt Foundation and by grants DFG 436, RFBR 03-02-17734, L.Sz. is partially supported by the French-Polish scientific agreement Polonium.

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